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Greenwich papers in political economy

Effective Demand and Say's Law in Marxist Theory: An Evolutionary Perspective

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Abstract

In this paper I theorize the roles of effective demand and Say's Law in the Marxist theory of exploitation and accumulation. I claim that an exogenous rate of exploitation implies deploying the strongest version of Say's Law, which leads profit rates not to equalize across sectors. Marx's own procedure in *Capital III* was therefore logically mistaken. Once Keynes' principle of effective demand is introduced, the rate of exploitation, and hence the distribution of income between wages and profits, becomes endogenous to aggregate demand. Profit rates can then equalize across sectors and prices of production can function as gravitational centers for market prices in a competitive economy. I develop an innovative evolutionary approach to demonstrate how effective demand, within the Marxist framework, determines the rate of exploitation and the rate of profit. At the intersection of Marx, Keynes, and Kalecki, my evolutionary framework integrates effective demand, functional income distribution, profit rate equalization, technological diffusion, and the gravitation towards prices of production.

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1. Introduction

The purpose of this paper is to theorize the roles of effective demand and Say's Law in the Marxist theory of exploitation and accumulation. I argue that an exogenous rate of exploitation implies deploying the strongest version Say's Law; which leads profit rates not to equalize across sectors. Once Keynes' principle of effective demand is introduced, the rate of exploitation, and hence the distribution of income between wages and profits, becomes endogenous to aggregate demand. Profit rates can then equalize across sectors and prices of production can function as gravitational centers for market prices in a competitive economy.

At the intersection of Marx, Keynes, and Kalecki, this paper develops an innovative evolutionary approach to demonstrate how aggregate demand, within the Marxist framework, determines the rate of exploitation, the rate of profit, the functional distribution of income, the diffusion of new techniques of production, the equalization of profitability, and the gravitation towards of prices of production. My approach allows for a clear contrast between Say's Law and Keynes' principle of effective demand in a Marxist framework. It additionally provides an evolutionary mechanism through which profit rates equalize under effective demand, which is a mechanism that Kalecki overlooked in his work. Lastly, my approach demonstrates that the Okishio theorem fails to hold once the real wage is endogenous.

In the Marxist tradition Say's Law has often appeared in its strongest version, which I shall call *Say's super-Identity* —that *all* values must be fully realized for *all* commodities produced. Say's super-Identity is usually introduced via the assumption that the rate of exploitation at the firm level is given exogenously. Under an exogenous rate of exploitation, the *ex ante* rate of exploitation in the production sphere is identical to the *ex post* rate of exploitation realized in the market.

Marx himself was ambiguous in his original writings regarding the role of effective demand (Robison 1947; Shoul 1957). In the three volumes of *Capital*, in the *Grundrisse*, and in *Theories of Surplus*

Value, he repeatedly claimed that there is no guarantee that markets will realize the values created in the sphere of production. Marx even located the potential lack of demand for the output in the function of money as a means of hoarding, of money as an end in itself. The Marxist literature then developed its own branch of inquiry into realization problems and realization crises. Yet, Marx also often assumed the strongest version of Say's Law in his reproduction schemes, such that all values produced were always fully realized. In the third volume of *Capital*, Marx introduced competition among different companies and offered novel ideas on profit rate equalization and falling profitability amid technological progress. But in this reproduction model the strongest version of Say's Law is again deployed and the demand side plays a secondary role in the theory of accumulation and exploitation.

Because of his death in 1883, Marx left unfinished the drafts of the second and third volumes of *Capital*. Engels later edited and published the manuscripts in the 1890s, but the connections between effective demand, exploitation, and accumulation were left incomplete. Since the advent of Keynesian and Kaleckian macroeconomics in the 1930s, Marxists have attempted to offer new insights into how the theory of effective demand relates to Marx's theory of capital accumulation. The influential works of Baran and Sweezy, for example, represented serious attempts to integrate aggregate demand into the Marxist framework. Dutt (2011), however, recently argued that the demand side in Marxist theory remains underdeveloped.

Marx's procedure in the third volume of *Capital* was logically inconsistent as he assumed simultaneously an exogenous rate of exploitation and profit rate equalization. His assumption of an exogenous rate of exploitation at the firm level implicitly depends on the strongest version of Say's Law; as the rate of exploitation can be exogenous only if *all* values produced, hence all surplus value and profits, are always fully realized. Because in Marx's reproduction schemes the total wage bill is advanced at the beginning of the capital circuit, an exogenous rate of exploitation predetermines the amount of profits to be realized even at the firm level. Profitability is predetermined in such a way that firms' profit rates become unresponsive to the amount of capital advanced in each sector. An exogenous rate of exploitation further

implies that the functional distribution of income between wages and profits is also exogenous and unresponsive to the accumulation of capital. In this case, there is no self-correction mechanism that would equalize profit rates across sectors. Prices of production, or the prices that correspond to equalized profit rates, cannot function as gravity centers for market prices.

However, if the principle of effective demand is introduced then Say's super-Identity no longer holds. In this case, as I will demonstrate, the rate of exploitation can become endogenous and dependent on aggregate demand, and profit rates can equalize across sectors. When aggregate demand is taken into account the *ex post* rate of exploitation automatically becomes dependent upon it and profit rates can equalize across sectors. This equalization occurs because sector profitability can then respond negatively to the capital committed to production in each sector. Over-supply in one sector will erode profits in that sector and capital will then gradually move to other sectors. Once the *ex post* rate of exploitation is endogenous to effective demand, the distribution of income between wages and profits also becomes endogenous to aggregate demand.

Marxist scholars must drop Say's Law once and for all. Incorporating the principle of effective demand into Marxist theory leads us to a better understanding of how aggregate demand determines the rate of exploitation, the rate of profit, and the gravitation towards prices of production.

2. Say's Law and Effective Demand

Becker and Baumol (1952) and Baumol (1977; 1999) were the first to notice the ambiguity of the term 'Say's Law'. This ambiguity is present in J. B. Say's own work and further reproduced by James Mill, David Ricardo, Marx, Keynes, Oskar Lange, and also by Kalecki. Becker and Baumol pointed out that Say's Law actually has two versions, one stronger (Say's Identity) and one weaker (Say's Equality). They proposed the following definitions:

- (i) *Walras' Law*: Total demand (including the demand for money) equals total supply (including the supply of money). This is just a definition; no direction of causality is implied between supply and demand.
- (ii) *Say's Identity (stronger version)*: Total supply automatically becomes, and is identical to, total demand. This happens because no one ever wants to hold cash, so all sales incomes are immediately spent on other goods and services. The demand for money does not affect aggregate demand, supply, or income. Money is just a veil. Recessions and cycles can happen but are entirely supply-driven.
- (iii) *Say's Equality (weaker version)*: Demand is supply-led and the equilibrium between aggregate demand and aggregate supply is stable, such that deviations from it are possible but self-correcting. Because of supply-side issues such as coordination problems and miscalculations, recessions and cycles are possible though brief. Money can be used as a store of value, and as a medium of exchange the supply of money is determined endogenously.

Growth, cycles, and recessions were supply-driven phenomena in Say's and Ricardo's reasoning, which represented the core of Classical Political Economy. Ricardo, in fact, claimed in many of his writings that cycles are not caused by aggregate demand deficiency but in fact by miscalculations about what to produce and in what proportions. The crucial issue is therefore not a lack of aggregate demand but a temporary mismatch between the composition of aggregate demand and aggregate supply, solved through the movement of capitals across sectors. Say and Ricardo attributed economic crisis not to oversupply but to underproduction (Béraud and Numa 2018; Kates 1997; Becker and Baumol 1952; Baumol 1977, 1999; Vianello 1989).

Applications of Say's Law are ambiguous because economists do not properly differentiate between Say's Identity and Say's Equality. In the Marxist tradition the situation is complicated by the fact that Marx himself had a dual position regarding Say's Law:

Marx evidently failed to realize how much the orthodox theory stands or falls with Say's Law and set himself the task of discovering a theory of crises which would apply to a world in which Say's Law was fulfilled, as well as the theory which arises when Say's Law is exploded. This dualism implants confusion in Marx's own argument, and, still more, in the arguments of his successors (Robinson 1947, p.51).

Marx certainly did not endorse Say's Law in any way in any of its versions but this duality is indeed part of his work (Shoul 1957). Despite criticizing Say's Law since the beginning of volume I of *Capital*, Marx often assumed its strongest version (what I call *Say's super-Identity*) in his reproduction schemes in volumes II and III, implying that values were always fully realized for all commodities produced.

Say's Law and Keynes' principle of effective demand determine in very different ways the direction of causality between the core elements of Marxist theory (Trigg 2006). An example of this is the direction of causality between aggregate demand and the profit rate. A substantial branch of the Marxist tradition would assign the profit rate as the cause and demand as the effect, which amounts to deploying Say's Law and making the economy supply-led. Discussions of the tendency of the profit rate to fall feature this type of reasoning. For underconsumptionists, on the contrary, the profit rate is the effect and aggregate demand is the cause, in which case the profit rate becomes itself endogenous to demand.

The principle of effective demand remains a hotly debated concept even within the post-Keynesian tradition. In this paper I follow Chick (1983), Hayes (2007), Hartwig (2007), Allain (2009), and Casarosa (1981) in their understanding of effective demand as the firms' effective commitment to production. Given the technology and cost structure, effective demand refers to how expected profitability determines supply and employment decisions at the *beginning* of the production period. Hence, this *ex ante* commitment is not identical to the *ex post* aggregate expenditures with consumption and investment. Effective demand is the firms' profit-maximizing expected proceeds, an *ex ante* concept relating to expectations but revised in line with *ex post* realized incomes. Even though aggregate demand and aggregate expenditure are not identical, they can be equal when *ex ante* expectations are fulfilled *ex post*, but not otherwise. Therefore,

“effective demand is an unfortunate term, for it really refers to the output that will be supplied; in general there is no assurance that it will also be demanded” (Chick 1983, p.65).

In Kalecki’s work effective demand is featured, but is not identical to that of Keynes. Assuming workers do not save, the capitalists’ aggregate expenditures with investment and consumption goods determine the level of aggregate profits. Workers spend what they get, while capitalists get what they spend. Real aggregate gross profit is determined entirely at the macro level, while income and output are set in the interaction between the micro and macro levels (Kriesler 1989). Kalecki derives his theory of effective demand from the national account identities and from the fact that capitalists decide their expenditures but not their incomes. As in Keynes, investment is autonomous from savings, and changes in income ensure that savings accommodate to the level of investment.

In Marxist theory the principle of effective demand means that the expected profit rate determines the firms’ constant and variable capitals advanced at the *beginning* of the production period, as well as the firms’ *supply* of commodities in the current production period. Market prices can change during the production period, but output changes only in the transition from one production period to the next. The beginning-of-period expenditures, which comprise the firms’ effective commitment to production, will then realize the values created at the *end of the previous* production period. Since the economy is structured as a chain of capital circuits, the *ex ante* aggregate demand at time $t+1$ determines the *ex post* realization of values from period t .

In the next section I discuss how effective demand determines the *ex post* rate of exploitation, and how it connects to the equalization of profit rates in the formation of prices of production.

3. Exploitation, Profit Rate Equalization, and Production Prices

In Chapter 9 of *Capital III*, Marx theorized the equalization of profit rates and the formation of prices of production. As I will demonstrate, Marx’s procedure was logically inconsistent, for he employed

an exogenous rate of exploitation and, hence, deployed Say's super-Identity. In Chapter 15 of *Capital III*, Marx then explicitly considered the difference between the *ex ante* and *ex post* rates of exploitation, mentioning the role of aggregate expenditures (consumption plus investment) in the realization of exploitation. Marx's insights in Chapter 15 actually offer a solution to his own mistake in Chapter 9:

The conditions for *immediate exploitation* and for the *realization of that exploitation* are not identical. Not only are they separate in time and space, they are also separate in theory. The former is restricted only by the society's productive forces, the latter by the proportionality between the different branches of production and by the society's power of consumption. And this is determined ... by the power of consumption within a given framework of antagonistic conditions of distribution [...]. It is further restricted by the drive for accumulation, the drive to expand capital and produce surplus-value on a larger scale (Marx [1894]1994, p.352-353 – emphasis added).

In the 1930s, Kalecki built on Marx's insights to claim that real aggregate gross profit is determined at the macro level by the capitalists' aggregate expenditures. Assuming workers do not save, capitalists cannot realize more surplus value in the aggregate than their own expenditures (Sardoni 2009; 1989). No matter how large the *ex ante* rate of exploitation in the production sphere, the capitalist class can only realize an *ex post* rate of exploitation that makes total profits match its own expenditures. Kalecki, however, did not explain how profit rates would equalize across sectors and hence ignored prices of production in his analysis (Jossa 1989; Vianello 1989).

The gravitation of market prices toward prices of production has been the object of rigorous study in the Marxist literature. These studies offer a level of technical detail that is much more precise than the numerical and verbal examples that Marx offered in volumes II and III of *Capital*. In this literature there is an agreement on two main results: (i) profit rates do not always equalize and thus prices of production cannot always function as stable attractors for market prices (Harris 1972; Nikaido 1983, 1985; Flaschel and Semmler 1985; Boggio 1985, 1990; Kubin 1990; Duménil and Lévy 1999, 1995; Prado 2006); (ii)

market prices that ensure balanced reproduction across sectors are not necessarily the set of prices that can also ensure profit rate equalization (Cockshott 2017).

On the empirical side of the literature (Scharfenaker and Semieniuk 2017, Fröhlich 2013, and Farjoun and Machover 1983) there has been a growing consensus that profitability converges not to a single uniform profit rate but actually to a non-uniform statistical equilibrium distribution of profit rates. Shaikh (2016) shows that profit rates on *new* investment projects (what Keynes labeled the ‘marginal efficiency of capital’) tend to equalize over time.

In this paper I build on this existing scholarship and introduce an evolutionary framework that closely mimics Marx’s original insights on exploitation, accumulation, and technological progress in volume III of *Capital*. I develop this framework to contrast the implications of the principle of effective demand with those of Say’s Law. My approach consists of using replicator dynamics from evolutionary game theory to describe the competitive selection that occurs simultaneously at the micro intra-sector and at the macro inter-sector levels. The replicator dynamics describe an updating process with random interactions in which behaviors with higher payoffs proliferate. It is a useful device to mimic the competitive struggle for survival in natural and social environments, for it models the process of equilibration by tracking the results of individual interactions (Bowles 2006; Gintis 2009; Prado 2006, 2002). The proposed framework formalizes key aspects of Marx’s theory of accumulation and profitability in an adaptive system, in which agents control their actions but not the aggregate consequences of their individual decisions. Micro decisions produce macro outcomes that then feed back again into micro decisions.

First, I formalize the macro inter-sector competition through which the aggregate and growing monetary capital of an economy is continuously redistributed between two sectors: sector I produces the means of production and sector II produces the final consumption good. The continuous redirection of monetary capital between sectors takes place according to profit rate differentials. Second, I formalize the micro intra-sector competition in which individual firms within each sector compete against each other via

cost-reducing technical change. Innovations are gradually adopted based on profit rate differentials within sectors. Once the real wage is endogenous, the Okishio (1961) theorem does not hold and technical change can reduce profitability over the longer run.

I then provide two different closures for the model. In the first closure I use an exogenous rate of exploitation that amounts to deploying the strongest version of Say's Law, under which all values produced are always fully realized. This is the same closure that Marx employed in Chapter 9 of *Capital III*. But unlike Marx's supposition, prices of production do not function as gravitational centers for market prices under Say's super-Identity. In the second closure, on the contrary, I introduce the principle of effective demand and make the rate of exploitation, the profit rate, and the growth rate dependent upon the level of aggregate demand. Marx suggests this second closure in Chapter 15 of *Capital III*. In this case profit rates can equalize and market prices can converge toward production prices. For both closures I present computer simulations and an analysis of the evolutionary stability of the long-run equilibria.

4. Macro Inter-Sector Competition

The economy-wide circuit of monetary capital, which starts and ends with capital in the form of money, can be represented through the following aggregation:

$$M_t - C_t \begin{cases} LP \\ MP \end{cases} \dots P \dots C'_t - M'_t \quad (1)$$

An initial amount of money M_t purchases two types of commodities as inputs, C_t : labor power (LP) and means of production (MP). During the subsequent production phase (... P ...) labor power creates more value than its own. The difference between the value that labor power creates and the value of labor power itself is the surplus value. The total value of the gross output C'_t contains the new value added created by productive workers, plus the pre-existing value transferred from the means of production. The gross output exchanges for a sum of money represented by the aggregate gross expenditures M'_t . The extra value that

workers create and for which they receive no compensation is the basis for the gross profits $\Delta M_t = M'_t - M_t$ in the system.

The economy comprises two sectors, each producing a single type of output using both labor power and means of production. Sector I supplies a homogenous type of means of production. Sector II supplies a homogenous type of final consumption good. Economic events take place temporally, therefore, the overlap of any two consecutive circuits of the total monetary capital can be represented as follows:

$$M_t - C_t \left\{ \begin{matrix} LP \\ MP \end{matrix} \dots P \dots C'_t - M'_t \right. \quad (2)$$

$$M_{t+1} - C_{t+1} \left\{ \begin{matrix} LP \\ MP \end{matrix} \dots P \dots C'_{t+1} - M'_{t+1} \right.$$

The circuit at period t+1 formally repeats the circuit at period t. The crucial relation is then that between the total value realized M'_t at the end of period t and the total monetary capital M_{t+1} advanced at the beginning of period t+1. Because of its supply-led principle, Say's Law in any of its versions means that causality runs from M_t to M'_t and then to M_{t+1} . Say's super-Identity, the strongest version of Say's Law, further implies that all values produced in $\dots P \dots$ are always fully realized in M'_t , and that the value realized in M'_t is advanced in M_{t+1} . The principle of effective demand, on the contrary, implies that the direction of causality actually runs from the *ex ante* demand M_{t+1} at the beginning of period t+1 to the realization of the total value M'_t at the end of the previous period t.

There is no fixed capital in this economy, so non-labor inputs are circulating capital only. The means of production that enter as inputs in sectors I and II in period t are the previous output of sector I in period t-1. Technology is represented by a linear production structure with fixed coefficients and constant returns to scale. Using a_{ji} to indicate the quantity of input j per unit of output i , the matrix of input-output coefficients is:

$$A = [a_{ji}] = \begin{bmatrix} a_{11} & a_{12} \\ 0 & 0 \end{bmatrix} \quad \text{with } 0 \leq a_{ji} < 1 \quad (3)$$

Using l_i to indicate the quantity of labor hours per unit of output in sector i , $r_{i,t}$ to indicate the within-sector profit rate per unit of output, p_i to indicate the market price per unit of output, and w to indicate the money wage per work hour, then per unit of output we have $[p_{j,t-1}a_{ji} + wl_i](1 + r_{i,t}) = p_{i,t}$. For each sector the price system is:

$$\begin{aligned} [p_{1,t-1}a_{11} + wl_1](1 + r_{1,t}) &= p_{1,t} \\ [p_{1,t-1}a_{12} + wl_2](1 + r_{2,t}) &= p_{2,t} \end{aligned} \quad (4)$$

The first term inside the brackets on the left-hand side represents constant capital, and the second term represents variable capital or the value of labor power, both in money terms. Their summation $[p_{j,t-1}a_{ji} + wl_i]$ is the unit cost. Competition within each sector then simultaneously determines profit rates and prices.

Although the nominal wage per work hour w is exogenously given by the bargaining power between workers and capitalists, the real wage $\frac{w}{p_{2,t}}$, in terms of quantities of the consumption good produced in sector II, is determined endogenously. Workers get their money wages and spend it as they like, not bound to any real wage specified in terms of a bundle of goods. Labor supply and credit are assumed not to be binding constraints on growth.

In each sector there is a collection of several firms and each of them can switch between sectors depending on the average profitability $\bar{r}_{i,t}$. Capitalists commit their capitals to where they expect to profit the most. But once firms flow into a sector aiming at the prevailing $\bar{r}_{i,t}$ they will immediately and unintentionally alter this average profitability. Supposing a very large collection of firms in the economy, we can normalize the total number of firms to unity and then consider only the evolution of population shares, with $f_{1,t}$ representing the fraction committed to sector I and $f_{2,t}$ the fraction committed to sector II:

$$M_t = f_{1,t}M_t + f_{2,t}M_t = M_{1,t} + M_{2,t} \quad \text{with} \quad f_{1,t} + f_{2,t} = 1 \quad (5)$$

Outputs $x_{i,t}$ supplied by each sector are the sectoral monetary capitals advanced divided by the respective unit costs. Sectoral supply expands when more monetary capital is advanced in the sector at the beginning of the production period, and it contracts when capitalists withdraw their initial expenditures:

$$x_{i,t} = \frac{f_{i,t}M_t}{[p_{1,t-1}a_{1i} + wl_i]} = \frac{M_{i,t}}{[p_{1,t-1}a_{1i} + wl_i]} \quad (6)$$

Within each sector, $M'_{i,t}$ indicates the end-of-period gross expenditures or the valorized monetary capitals that comprise the original monetary capitals $M_{i,t}$ advanced plus the surplus value realized. Market prices $p_{i,t}$ are the end-of-period expenditures divided by quantities supplied:

$$p_{i,t} = \frac{M'_{i,t}}{x_{i,t}} = \frac{M_{i,t}(1 + \bar{r}_{i,t})}{x_{i,t}} = \frac{f_{i,t}M_t(1 + \bar{r}_{i,t})}{x_{i,t}} \quad (7)$$

The monetary capital $M_{i,t}$ committed to sector i at the beginning of time t is valorized on average to $(1 + \bar{r}_{i,t})$ after the output is sold. The fraction $(1 + \bar{r}_{i,t})$ includes the replication of the money initially spent plus average profits. Hence, the valorized capital in each sector is $M'_{i,t} = M_{i,t}(1 + \bar{r}_{i,t}) = f_{i,t}M_t(1 + \bar{r}_{i,t})$. Using \tilde{r}_t to indicate the economy-wide weighted average profit rate, such that $(1 + \tilde{r}_t) = \sum_i f_{i,t}(1 + \bar{r}_{i,t})$, the aggregate valorized capital for the entire economy is:

$$M'_t = M_t(1 + \tilde{r}_t) = \sum_i M_{i,t}(1 + \bar{r}_{i,t}) = \sum_i f_{i,t}M_t(1 + \bar{r}_{i,t}) \quad (8)$$

The shares of the total monetary capital advanced at the beginning of period $t+1$ change according to the average profitability obtained in period t in each sector:

$$f_{i,t+1} = \frac{M'_{i,t}}{M'_t} = \frac{f_{i,t} M_t (1 + \bar{r}_{i,t})}{M_t (1 + \tilde{r}_t)} = f_{i,t} \frac{(1 + \bar{r}_{i,t})}{(1 + \tilde{r}_t)} \quad (9)$$

Rewriting it as $\frac{f_{i,t+1}}{f_{i,t}} = \frac{(1 + \bar{r}_{i,t})}{(1 + \tilde{r}_t)}$, subtracting 1 from both sides and using $\Delta f_{i,t+1} = f_{i,t+1} - f_{i,t}$,

we then obtain the replicator equation that formalizes the macro competition between capitalists across sectors:

$$\Delta f_{i,t+1} = \mu f_{i,t} \left(\frac{1}{1 + \tilde{r}_t} \right) [\bar{r}_{i,t} - \tilde{r}_t] \quad (10)$$

The profitability gap in relation to the economy-wide average determines how capitalists allocate their monetary capitals across sectors. The coefficient $\mu \in (0,1]$ indicates that only a fraction of the capitalists in each sector will in fact shift their capital to a different activity that is currently benefitting from higher returns. The complementary fraction $(1 - \mu)$ of the firms cannot update their behavior even when return differentials are an incentive for them to do so.

The individual search for profits creates unintended consequences in both the sector-level and the economy-wide average profit rates. Capitalists make decisions based on profit rates prevailing in each sector, not on the economy-wide profit rate. But they end up affecting aggregate profitability through their decentralized individual actions to move their capitals from one sector to another. The effects on the aggregate profit rate then feed back into individual decisions about where to commit the monetary capital in the following period.

The equations so far presented describe the growth of output and the evolutionary adjustments that regulate the shares of monetary capitals over time at the macro level. In the next section I turn to the competition for profits through cost-reducing technical change that characterizes the micro-adjustments within each sector.

5. Micro Intra-Sector Competition

Large collections of firms compete for profits within each sector. Markets are intensely competitive, forcing firms to sell at prevailing market prices. The way to increase individual profit lies therefore with the adoption of new cost-reducing technologies. Innovations are generated exogenously and then adopted conditional on enhancing individual profitability. When an individual firm decides upon the adoption of a new productive structure it does so taking the prevailing market price as given. But the individual adoption of the newer technique changes the sector cost structure, and it therefore unintentionally affects the market price. The new market price then operates as a signal for the remaining firms to also adopt the cost-reducing technique. Each sector will thus display a production structure that is a combination of firms producing with the new technique and firms still producing with the old technique.

The economy has three evolutionary processes taking place concurrently. The first is the evolutionary diffusion of new techniques in the sector producing means of production. The second is the evolutionary diffusion of new techniques in the sector producing final consumption goods. The third is the evolutionary distribution of the growing aggregate monetary capital between sectors. An individual decision to adopt a new technique thus triggers a chain of reactions and feedback effects that no individual capitalist can anticipate. Externalities exist in this economy given that firms do not fully internalize the social consequences of their individual actions.

The prevailing technique of production is represented in the set of four technical parameters $(a_{11}^o, a_{12}^o, l_1^o, l_2^o)$. An innovation $(a_{11}^n, a_{12}^n, l_1^n, l_2^n)$ can imply the use of more of labor power and means of production, less of both inputs, or more of one input and less of the other (superscript o for ‘old’ and n for ‘new’ technique). Rearranging the price equations in (4) we get the profit rate per unit produced using current technology, $r_{i,t}^o$:

$$r_{i,t}^o = \frac{p_{i,t}}{[p_{1,t-1}a_{1i}^o + wl_i^o]} - 1 \quad (11)$$

Similarly, the profit rate associated with the new technology is:

$$r_{i,t}^n = \frac{p_{i,t}}{[p_{1,t-1}a_{1i}^n + wl_i^n]} - 1 \quad (12)$$

The evolutionary diffusion of a new technique can then be formalized with the dynamics of replication. The variable $v_{i,t} \in [0,1]$ indicates the share of firms in sector i that adopt the new technique at time t , while $(1 - v_{i,t})$ indicates the share that remains with the older technique. Because each sector has a large collection of firms, and assuming that they interact through random pairwise matching, we can use a simple replicator equation for the diffusion of innovations. Normalizing population sizes to unity allows us to work with population shares in each sector as follows:

$$\begin{aligned} v_{i,t+1} &= v_{i,t} + v_{i,t}(1 - v_{i,t})[r_{i,t}^n - r_{i,t}^o] \\ \Delta v_{i,t+1} &= v_{i,t}(1 - v_{i,t})[r_{i,t}^n - r_{i,t}^o] \\ \Delta v_{i,t+1} &= v_{i,t}[r_{i,t}^n - \bar{r}_{i,t}] \end{aligned} \quad (13)$$

The term $v_{i,t}(1 - v_{i,t})$ is the variance of the firms within each sector and the term $[r_{i,t}^n - r_{i,t}^o]$ is the differential replication selection, so that the updating process is payoff monotonic. The third line in equation (13) follows from the fact that the average profit rate in each sector is: $\bar{r}_{i,t} = (v_{i,t})[r_{i,t}^n] + (1 - v_{i,t})[r_{i,t}^o]$.

Technical change and its evolutionary diffusion imply that older and newer cost structures coexist until the newer technique completely replaces the older one. Given the monetary capital $M_{i,t}$ committed to each sector, the new quantities supplied can be found by dividing the monetary capital advanced by the mixed cost structure:

$$x_{i,t} = \frac{M_{i,t}}{(v_{i,t})[p_{1,t-1}a_{1i}^n + wl_i^n] + (1 - v_{i,t})[p_{1,t-1}a_{1i}^o + wl_i^o]} \quad (14)$$

The supply equation in (14) thus replaces the supply equation in (6), which only applied to production under a single technology. Average rates of profit in each sector now depend on the prevailing market prices and on the linear combination between older and newer techniques:

$$\bar{r}_{i,t} = \frac{p_{i,t}}{(v_{i,t})[p_{1,t-1}a_{1i}^n + wl_i^n] + (1 - v_{i,t})[p_{1,t-1}a_{1i}^o + wl_i^o]} - 1 = \frac{\Delta M_{i,t}}{M_{i,t}} \quad (15)$$

As soon as profit rates in each sector change from their previous position they trigger intra-sector competition via the micro replicator dynamic in equation (13) as well as inter-sector competition via the macro replicator dynamic in equation (10). The out-of-equilibrium adjustments and the evolution of the system over time explicitly reflect the interplay of unintended social consequences of uncoordinated individual actions. In the next section I analyze the stationary states that might prevail in the long run.

6. Long-Run Equilibria and Evolutionary Stability

In an evolutionary setting, differential replication offers a behavioral microfoundation for spontaneous and path-dependent interactions of multiple uncoordinated agents. The model becomes more intuitive if we focus on the trajectories of the three replicator equations $(f_{1,t}, v_{1,t}, v_{2,t})$ toward their long-run stationary states. Stationary states are those states at which the replicator reaches a fixed point with no further changes in the replication process $(\Delta f_{1,t} = 0, \Delta v_{1,t} = 0, \Delta v_{2,t} = 0)$. The crucial procedure is to know which strategies are going to prevail asymptotically when $t \rightarrow \infty$.

In an evolutionary game with replicator dynamics we know that the evolutionarily stable strategies prevail over the long run. An *evolutionarily stable strategy* (ESS) is a best response to itself and hence it is a symmetric Nash equilibrium that is also asymptotically stable in its respective replicator equation. *Evolutionary stability* implies both self-correction and asymptotic attractiveness (i.e., it is a stable attractor),

hence the system converges over time to a stationary point that is evolutionarily stable (Bowles 2006; Gintis 2009; Elaydi 2005; Scheinerman 2000).

In Table 1 I summarize the stationary states and asymptotic properties of each replicator equation. Note that in the macro inter-sector dynamic, when there is no ESS, the system converges to an interior stable solution f_1^* such that average profit rates are equalized asymptotically across sectors. In this case, profit rates are not just *equal* across sectors but truly *equalized* in the sense that the equality in sector profitability is evolutionarily stable. In the Appendix, I provide a formal stability analysis of the stationary states.

[Table 1 about here]

Because the technical coefficients in the input-output matrix are exogenous but not constant, a strategy that was an ESS *before* the technical change might not be an ESS *after* the innovation is introduced. As long as we have exogenous innovations brought into the system, the ESS's themselves will change over time.

The closure imposed on the system will determine which long-run equilibrium will prevail and whether or not the stationary state will be stable. In the next section I analyze the implications of an exogenous rate of exploitation as a first closure. In the subsequent section I then analyze an alternative closure with endogenous rates of exploitation.

7. Say's Law and the Exogenous Rate of Exploitation

The first closure consists of an exogenous rate of exploitation and is the same closure that Marx employed in Chapter 9 of *Capital III*. An exogenous rate of exploitation means that the *ex ante* rate of

exploitation in production is identical to the rate of exploitation realized *ex post* in the market. An exogenous rate of exploitation, however, implies the deployment of Say's super-Identity, the strongest version of Say's Law.

When the rate of exploitation is predetermined at the firm level then all values produced must be fully realized for *all* commodities, otherwise the rate of exploitation would not be constant throughout the circuit of capital. If we were to consider realization problems associated with the lack of effective demand, or if we were simply to acknowledge the fact that aggregate demand determines incomes, the rate of exploitation could not be predetermined exogenously at any given level.

Because all values must be fully realized, and given that in the circuit of capital the total expenditures on labor power and means of production take place at the *beginning* of each production period, an exogenous rate of exploitation is equivalent to an exogenous functional distribution of income between wages and profits. The wage share in value added is $\frac{V}{V+S} = \frac{1}{1+e}$, in which V is the value of labor power (the total wage bill), S is surplus value or profits, $e = \frac{S}{V}$ is the rate of exploitation, and V+S is the flow of value added in the economy. When the rate of exploitation is exogenous it will automatically predetermine the wage and profit shares of national income. The functional distribution of income is fixed and thus unresponsive to capital accumulation.

In neo-Kaleckian models (Dutt 1990, 1984; Marglin 1984; Badhuri and Marglin 1990) the markup is exogenous and prices are fixed per unit of output; thus income distribution between wages and profits is also exogenous. Effective demand then determines the level of aggregate output and income. But this is not the case in Marx's circuit of capital, as the beginning-of-period aggregate expenditures on wages and means of production are already set at their nominal levels. If wages vary according to the level of revenues realized and are paid not at the beginning of the circuit but at the end, the rate of exploitation becomes endogenous to demand. In the circuit of capital we can have either an exogenous rate of exploitation or the principle of effective demand at play, but not both simultaneously.

Once the rate of exploitation $e = \frac{S}{V}$ is given exogenously at the firm level, and as V is fixed at the beginning of the capital circuit, this predetermines the amount of surplus value, S , to be realized and thus prevents individual profit rates from equalizing. The economy has no self-correction mechanism that would make profit rates sensitive to the amount of monetary capital advanced in each sector. As profit rates do not equalize in the long run, prices of production cannot function as gravitational centers for market prices. In the lines that follow I develop this reasoning in more technical detail.

The hours worked per unit of output, l_i , generate the value added that corresponds to the summation of wages and profits. Workers in sector i produce $wl_i(1 + e)$ of value added per unit of output but only get back the value of their labor power corresponding to wl_i , thus leaving the surplus ewl_i to the capitalists hiring them. The rate of exploitation e is fixed and equal for all firms. The end-of-period expenditures match the total value produced, and profits originate from unpaid labor time:

$$M'_{i,t} = [p_{1,t-1}a_{1i} + wl_i(1 + e)]x_{i,t} \quad (16)$$

$$\Delta M_{i,t} = M'_{i,t} - M_{i,t} = e wl_i x_{i,t} \quad (17)$$

In *qualitative* terms, profits originate from surplus value. In *quantitative* terms, and in this closure, causality runs from exploitation to profits ($e \rightarrow \Delta M_{i,t}$). Because of its supply-led principle, any version of Say's Law implies that the quantity of surplus value produced determines the amount of surplus value realized as profits. This particular relation between profitability and exploitation derives from the fact that the price system is such that $p_{i,t} = p_{1,t-1}a_{1i} + wl_i + ewl_i = [p_{1,t-1}a_{1i} + wl_i](1 + r_{i,t})$. Rearranging terms and solving for the profit rate gives us:

$$r_{i,t} = \frac{e}{1 + \left(\frac{p_{1,t-1}}{w}\right)\left(\frac{a_{1i}}{l_i}\right)} \quad (18)$$

Equation (18) is the usual Marxist relation in which the profit rate is the rate of exploitation divided by one plus the organic composition of capital. The organic composition is, in turn, the relative price $\frac{p_{1,t-1}}{w}$ times the technical composition $\frac{a_{1i}}{l_i}$ between constant and variable capital.

When e is fixed, the long-run $r_{i,t}$ will also be predetermined in both sectors as w , a_{1i} , and l_i are all parameters, and Say's super-Identity in sector I also predetermines the path of $p_{1,t}$. As $0 \leq a_{11} < 1$, there is a stationary state such that $p_{1,t} \rightarrow \frac{wl_1(1+e)}{1-a_{11}}$ as $t \rightarrow \infty$. However, even though market prices converge to a stationary state, this stationary state is not a price of production as profit rates are not equalized across sectors.

Once firms begin to adopt technological innovations, the mixed productive structure requires weighting the surplus value produced by the respective shares of firms employing the newer and older technologies. Equations (19) and (20) replace equations (16) and (17) as soon as a new technique is introduced:

$$M'_{i,t} = \{(v_{i,t})[p_{1,t-1}a_{1i}^n + wl_i^n(1+e)] + (1-v_{i,t})[p_{1,t-1}a_{1i}^o + wl_i^o(1+e)]\}x_{i,t} \quad (19)$$

$$\Delta M_{i,t} = M'_{i,t} - M_{i,t} = \{(v_{i,t})[wl_i^n e] + (1-v_{i,t})[wl_i^o e]\}x_{i,t} \quad (20)$$

Increments in the share of firms adopting the new technology ($v_{i,t}$) can reduce or increase the average profit rate prevailing in a sector. But the final effect on profitability can only be known after the repricing of both the means of production and the final consumption good.

To simulate the model in its first closure it is necessary to fix eleven parameters and three initial conditions. In this example the initial technical coefficients are set to $(a_{11}^o, a_{12}^o, l_1^o, l_2^o) = (0.2, 0.1, 0.7, 0.7)$ for the old technology. The nominal wage w is set to 10 dollars per work hour, the rate of exploitation e is set to 110%, and only $\mu=20\%$ of the firms migrate to another sector according to inter-sector average profitability differentials. The initial aggregate monetary capital $M_{t=1}$ is set to 100 dollars, and the initial

distribution is set at 60% to sector I ($f_{1,t=1} = 0.6$) and 40% to sector II ($f_{2,t=1} = 0.4$). The means of production are initially priced at 50 dollars per unit ($p_{1,t=0} = 50$). As I show in the Appendix, the long-run stationary state is independent from these arbitrary initial conditions.

The model is set to run for 400 periods. For the first 49 rounds the trajectories evolve without technical change. At period $t=50$, I introduce an innovation in sector II that increases labor productivity by 100% while increasing the use of machines by 100% per unit of output, hence $(a_{11}^o, a_{12}^n, l_1^o, l_2^n) = (0.2, 0.2, 0.7, 0.35)$. This machine-intensive labor-saving innovation generates a strong increase in the technical composition of capital in the sector producing the consumption good. At time $t=100$, I introduce an innovation in sector I that increases labor productivity by 150% and the use of the machines by 100% per unit of output such that $(a_{11}^n, a_{12}^n, l_1^n, l_2^n) = (0.4, 0.2, 0.28, 0.35)$. This innovation implies a strong machine-intensive labor-saving technical change in the sector producing the means of production.

[Figure 1 about here]

In Figure 1 I report the evolution of key variables under this first closure of the model. As expected, profit rates do not equalize even though they are temporarily equal across sectors for two specific moments in time, during the diffusion of the new techniques of production. Therefore, prices of production cannot function as gravitational centers for market prices. And even though the real wage is determined endogenously, the exogenous rate of exploitation fixes constant profit and wage shares of value added.

The simulation additionally shows that the uncoordinated implementation of the new technologies increases the profit rate only for those firms initially adopting the innovation, but the gradual diffusion of the new technologies results in lower levels of profitability for all capitalists over time. Because the real wage is endogenous, not exogenous as in Okishio's (1961) theorem, technical change can reduce the profit rate over the longer run.

8. Effective Demand and the Endogenous Rate of Exploitation

In this section I offer a second closure in which the *ex post* rate of exploitation is endogenous and dependent upon the level of aggregate demand, as Marx suggested in Chapter 15 of *Capital III*. Once effective demand is brought into the framework, the rate of exploitation, and with it the distribution of income between wages and profits, automatically becomes dependent upon the demand side. Since effective demand determines the amount of surplus value realized and the *ex post* rate of exploitation, profitability becomes sensitive to the amount of monetary capital committed to each sector. Profit rates then equalize as long as $\frac{d\bar{r}_{1,t}}{df_{1,t}} < \frac{d\bar{r}_{2,t}}{df_{1,t}}$. In the Appendix I show under what parameter values this condition is met.

The monetary capital $M_{1,t+1}$ effectively committed to production in sector I at the beginning of period t+1 reflects the capitalists' expected profitability in that sector. This monetary capital $M_{1,t+1}$ advanced is the *ex ante* demand at the beginning of period t+1, and as such it comprises the expenditure $M'_{1,t}$ necessary to realize the value produced in sector I at the end of the previous period, t.

In this closure I opt for the neo-Keynesian autonomous investment function à la Joan Robinson (1962; see also Dutt 2011; 1990; Marglin 1984) which assumes that firms operate at full capacity utilization and that the amount of monetary capital committed to the investment good sector is a function of past profitability. The parameters γ_i indicate the sensitivity of *ex ante* investment demand to the observed profit rates in each sector, and the autonomous component is simply the investment carried out in the previous period ($M'_{1,t-1}$). Given that there are firms operating with the newer and older technologies simultaneously in each sector, we have that:

$$M_{1,t+1} = M'_{1,t} = M'_{1,t-1} + \gamma_1 \{ (v_{1,t}) [r_{1,t-1}^n] + (1 - v_{1,t}) [r_{1,t-1}^0] \} M_{1,t-1} \quad (21)$$

$$+ \gamma_2 \{ (v_{2,t}) [r_{2,t-1}^n] + (1 - v_{2,t}) [r_{2,t-1}^0] \} M_{2,t-1}$$

In sector II, likewise, the monetary capital $M_{2,t+1}$ effectively committed to production at the beginning of period t+1 reflects the capitalists' expected profitability for that sector. Supposing that workers do not save and that there is no consumption credit, the total expenditure $M'_{2,t}$ with the consumption goods produced in sector II is simply the total wage bill in the economy. At the beginning of period t+1, capitalists commit to sector II an amount of monetary capital proportional to the aggregate consumption of out wages realized in the previous production period, t. Given that the wage bills in each sector must be weighted by the shares of firms using the old and the new technologies, we have that:

$$M_{2,t+1} = M'_{2,t} = \{(v_{1,t})[wl_1^n] + (1 - v_{1,t})[wl_1^0]\} x_{1,t} + \{(v_{2,t})[wl_2^n] + (1 - v_{2,t})[wl_2^0]\} x_{2,t} \quad (22)$$

Therefore, effective demand at the beginning of period t+1 is $M_{t+1} \equiv M_{1,t+1} + M_{2,t+1} = M'_{1,t} + M'_{2,t}$, in which the second equality follows directly from equations (5) and (9). The endogenous rates of exploitation $e_{i,t}$ within each sector are the sector surplus values realized over the nominal wage bill advanced:

$$e_{i,t} = \frac{M_{i,t+1} - M_{i,t}}{\{(v_{i,t})[wl_i^n] + (1 - v_{i,t})[wl_i^0]\} x_{i,t}} = \frac{M'_{i,t} - M_{i,t}}{\{(v_{i,t})[wl_i^n] + (1 - v_{i,t})[wl_i^0]\} x_{i,t}} \quad (23)$$

The rates of exploitation in each sector depend directly on the level of aggregate demand from equations (21) and (22). As under Say's Law, in *qualitative* terms, profits originate from surplus value. But under Say's Law, in *quantitative* terms, the determination ran from exploitation to realized profits ($e \rightarrow \Delta M_{i,t}$). The principle of effective demand, on the contrary, now implies that in *quantitative* terms the determination runs from profits to realized exploitation ($e_{i,t} \leftarrow \Delta M_{i,t}$). Even though profits originate *qualitatively* from surplus value, under the principle of effective demand the amount of profits is the *quantity* of surplus value actually realized.

I simulate the model in its second closure to illustrate these points. To facilitate comparison with the model in its first closure I keep the same parameter values and innovation patterns as in the previous simulation. For the investment function I set $\gamma_1 = \gamma_2 = 0.5$, and investment demand begins at 50 dollars ($M'_{1,t=1} = 50$) as an initial condition. Simulation results for key variables are reported in Figure 2.

[Figure 2 about here]

Contrary to the first closure, the principle of effective demand can make profit rate equalization an evolutionarily stable long-run equilibrium. Prices of production can thus operate as gravitational centers for market prices. The level of exploitation, the real wage, as well as the wage and profit shares of value added now all respond to the trajectory of aggregate demand. As in the first closure, Okishio's (1961) theorem does not hold when the real wage is endogenous: technical change increases the profit rates of the early adopters but ultimately reduces profitability over time for all companies in both sectors.

9. Conclusion

Marx was logically inconsistent in Chapter 9 of *Capital III* when he employed an exogenous rate of exploitation together with equalizing profit rates. Because an exogenous rate of exploitation implies the deployment of Say's super-Identity —the strongest version of Say's Law— profit rates cannot equalize across sectors and production prices cannot operate as gravitational centers for market prices. My solution to this logical inconsistency in Marx's approach is to introduce Keynes' principle of effective demand; such that the rate of exploitation becomes endogenous to the level of aggregate demand. Marx hinted at this solution in Chapter 15 of *Capital III*. The principle of effective demand allows for profit rates to equalize across sectors and prices of production to operate as stable attractors to market prices.

In this paper I developed an innovative evolutionary approach that formalizes Marx's model of a competitive economy with technical change. I employed replicator dynamics and evolutionary game theory to demonstrate the superiority of Keynes' principle of effective demand over Say's Law within the Marxist framework of exploitation and accumulation. My evolutionary approach offers a clearer and more precise presentation of Marx's system in *Capital III*. My approach additionally provides a way to integrate the equalization of profit rates within Kalecki's framework, and I demonstrate that the Okishio theorem does not hold if the real wage is endogenous.

Marxist scholars must drop Say's Law if we aim to develop a theory that is both empirically relevant and logically consistent. The principle of effective demand offers us a better understanding of how aggregate demand determines the rate of exploitation, the rate of profit, the functional distribution of income, the diffusion of technological innovation, and the gravitation toward prices of production.

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Appendix: Stability Analysis

In this Appendix I present the stability analysis of the long-run stationary states under both closures. To avoid unnecessary complications I suppose no technical change in either sector ($v_{i,t} = 0$ or 1) and that the updating share is 100% ($\mu = 1$).

Asymptotic stability means that the stationary state is both *stable* and an *attractor*, so the system converges to it over time (Scheinerman, 2000; Elaydi, 2005). In the one-dimensional replicator equation, asymptotic stability requires the payoff of a strategy to increase less than the competing payoff when the agents adopting that strategy increase their share in the population (Bowles 2006; Gintis 2009). The expected payoffs are the average profit rates within each sector; thus the stability condition is:

$$\frac{d\bar{r}_{1,t}}{df_{1,t}} < \frac{d\bar{r}_{2,t}}{df_{1,t}} \quad (\text{A.1})$$

Under an exogenous rate of exploitation the model has a stationary state at $p_1^* = \frac{wl_1(1+e)}{1-a_{11}}$; $p_2^* = \left(\frac{a_{12}}{1-a_{11}}l_1 + l_2\right)w(1+e)$; $r_1^* = \frac{e}{\left(\frac{a_{11}}{1-a_{11}}\right)(1+e)+1}$; and $r_2^* = \frac{e}{\left(\frac{a_{12}l_1}{1-a_{11}l_2}\right)(1+e)+1}$. Profit rates are given in equation (19) and $\frac{dr_{1,t}}{df_{1,t}} = 0$ and $\frac{dr_{2,t}}{df_{1,t}} = 0$; thus the stability condition (A.1) is never satisfied. As described in Table

1, three long-run equilibria are possible in this case:

- (i) The technical composition of capital is the same across sectors $\left(\frac{a_{11}}{l_1} = \frac{a_{12}}{l_2}\right)$, which implies that $r_1^* = r_2^*$ and $f_1^* = f_2^* = 0.5$. The model is always at the unstable equilibrium.
- (ii) The technical composition of capital is higher in sector II $\left(\frac{a_{11}}{l_1} < \frac{a_{12}}{l_2}\right)$, which implies that $r_1^* > r_2^*$ and all monetary capital flows over time to sector I ($f_1^* = 1$).
- (iii) The technical composition of capital is higher in sector I $\left(\frac{a_{11}}{l_1} > \frac{a_{12}}{l_2}\right)$, which implies that $r_1^* < r_2^*$ and all monetary capital flows over time to sector II ($f_1^* = 0$).

Under an endogenous rate of exploitation the model has a stationary state with equalized profit

rates across sectors at $p_1^* = \frac{wl_1(1+r^*)}{1-a_{11}(1+r^*)}$; $p_2^* = \left[\frac{wl_1a_{12}(1+r^*)}{1-a_{11}(1+r^*)} + wl_2 \right] (1+r^*)$; $\left(\frac{e_1}{e_2} \right)^* = \frac{\frac{p_1^* a_{11} + 1}{w \cdot l_1}}{\frac{p_1^* a_{12} + 1}{w \cdot l_2}}$; $\left(\frac{x_1}{x_2} \right)^* = \left(\frac{f_1^*}{1-f_1^*} \right) \frac{p_1^* a_{12} + wl_2}{p_1^* a_{11} + wl_1}$. With some algebraic manipulation, the stability condition (A.1) is satisfied when:

$$\gamma_1 \left(\frac{f_{1,t-1}}{1 + \tilde{r}_{t-1}} \right) - \gamma_2 \left(\frac{f_{1,t-1} + \tilde{r}_{t-1}}{1 + \tilde{r}_{t-1}} \right) < \left(\frac{f_{1,t}}{1 - f_{1,t}} \right)^2 \left(\frac{1}{\frac{p_{1,t-1} a_{11}}{w} + 1} \right) \quad (\text{A.2})$$

This inequality means that the stationary state $0 < f_1^* < 1$ tends to become less stable when, *ceteris paribus*: (i) the organic composition of capital in sector I $\left(\frac{p_{1,t-1} a_{11}}{w} \right)$ is very high; (ii) the ratio between the sector investment coefficients $\left(\frac{\gamma_1}{\gamma_2} \right)$ is very high. Computer simulations confirm these results and further indicate that the stationary state becomes less stable, *ceteris paribus*, at low values of the updating share μ . But as long as γ_1 and γ_2 are not too far apart and the technical composition $\frac{a_{11}}{l_1}$ in sector I is not too high, the stationary state with equalized profit rates is asymptotically stable under the principle of effective demand.

The stability condition (A.1) refers to the one-dimensional replicator in which the economy has only two sectors, so the interior solution $0 < f_1^* < 1$ is either stable or unstable. In an economy with three or more sectors the stability condition in higher dimensions would cover cases with saddle path stability and limit cycles. This issue is beyond the scope of this paper but will be pursued in further work.

Tables and Figures

Table 1: Stationary States and Asymptotic Properties

Stationary States			
(a) Macro Inter-Sector Replicator			
Sector I is ESS Sector II is ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} \geq \bar{r}_{2,t}$	$f_{1,t} = 1$ is stable $f_{1,t} = 0$ is stable $0 < f_{i,t}^* < 1$ is unstable
Sector I is ESS Sector II is not ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} > \bar{r}_{2,t}$	$f_{1,t} = 1$ is stable $f_{1,t} = 0$ is unstable
Sector I is not ESS Sector II is ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} < \bar{r}_{2,t}$	$f_{1,t} = 1$ is unstable $f_{1,t} = 0$ is stable
No ESS	$\Delta f_{i,t+1} = 0$	$\bar{r}_{1,t} = \bar{r}_{2,t}$	$0 < f_{i,t}^* < 1$ is stable $f_{1,t} = 1$ is unstable $f_{1,t} = 0$ is unstable
(b) Micro Intra-Sector Replicator in Sector I			
Innovate is ESS Not innovate is ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^n \geq r_{1,t}^o$	$v_{1,t} = 1$ is stable $v_{1,t} = 0$ is stable $0 < v_{1,t}^* < 1$ is unstable
Innovate is ESS Not innovate is not ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^n > r_{1,t}^o$	$v_{1,t} = 1$ is stable $v_{1,t} = 0$ is unstable
Innovate is not ESS Not innovate is ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^n < r_{1,t}^o$	$v_{1,t} = 1$ is unstable $v_{1,t} = 0$ is stable
No ESS	$\Delta v_{1,t+1} = 0$	$r_{1,t}^n = r_{1,t}^o$	$0 < v_{1,t}^* < 1$ is stable $v_{1,t} = 1$ is unstable $v_{1,t} = 0$ is unstable
(c) Micro Intra-Sector Replicator in Sector II			
Innovate is ESS Not innovate is ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^n \geq r_{2,t}^o$	$v_{2,t} = 1$ is stable $v_{2,t} = 0$ is stable $0 < v_{2,t}^* < 1$ is unstable
Innovate is ESS Not innovate is not ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^n > r_{2,t}^o$	$v_{2,t} = 1$ is stable $v_{2,t} = 0$ is unstable
Innovate is not ESS Not innovate is ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^n < r_{2,t}^o$	$v_{2,t} = 1$ is unstable $v_{2,t} = 0$ is stable
No ESS	$\Delta v_{2,t+1} = 0$	$r_{2,t}^n = r_{2,t}^o$	$0 < v_{2,t}^* < 1$ is stable $v_{2,t} = 1$ is unstable $v_{1,t} = 0$ is unstable

Figure 1: Simulation of the Evolutionary Model with Say's super-Identity and an Exogenous Rate of Exploitation

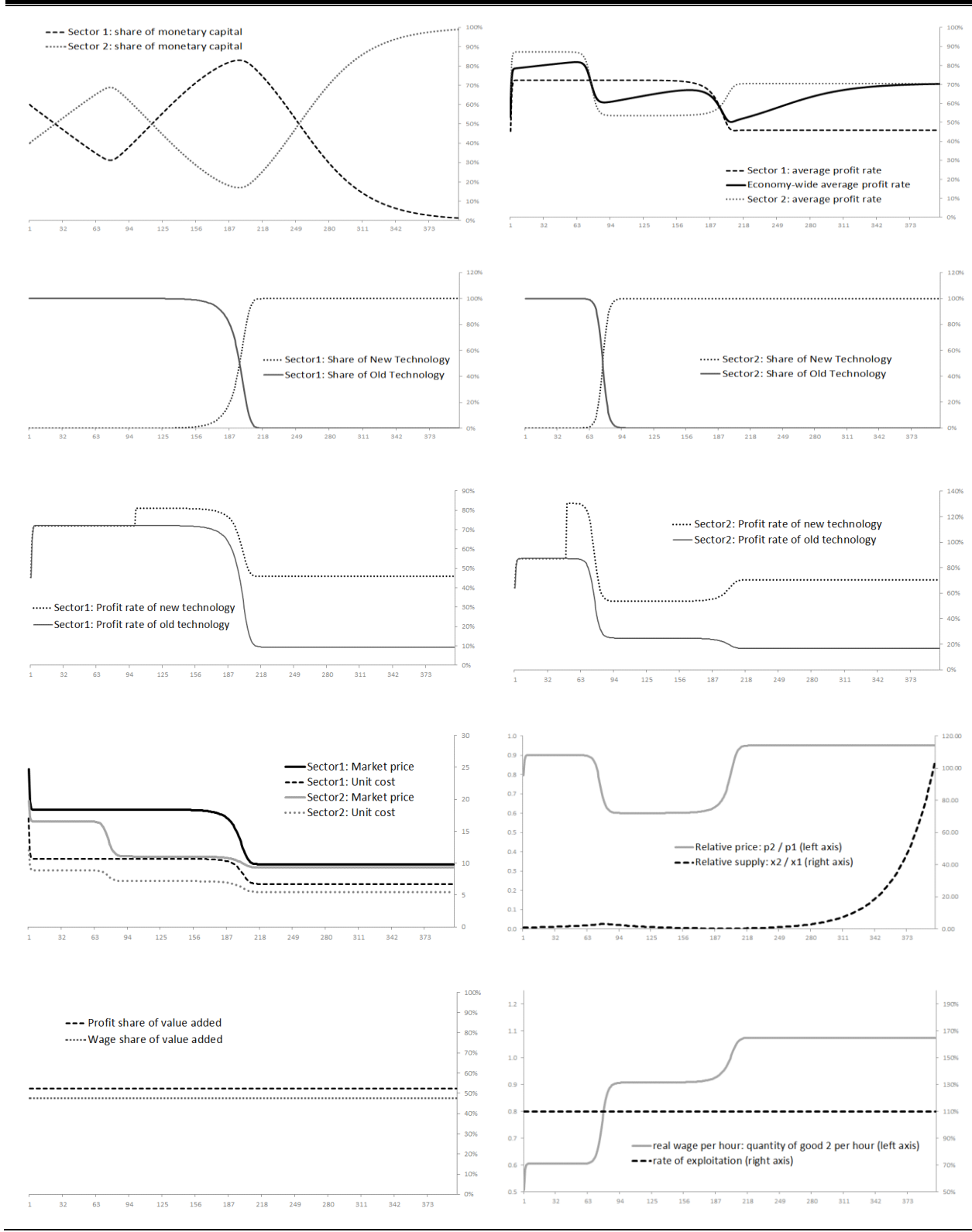


Figure 2: Simulation of the Evolutionary Model with Effective Demand and Endogenous Rates of Exploitation

