# INSIDE/OUTSIDE-TIME: METABOLAE IN IANNIS XENAKIS'S TETORA 

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#### Abstract

This paper is an analytical approach to Xenakis's 1990 string quartet Tetora. The analysis reflects the second phase of the application of sieve theory. Xenakis began developing his theory in 1963, when he was named artist in residence in West-Berlin. His studies, at that time, led him to the quest of an axiomatics of musical structures which can be built through highly formalised procedures. This is the case of Nomos Alpha (1966), but not with works from his later period. This paper touches upon the relation that the composer had developed with his own theory in its relocation (mid 1980s - mid 1990s). Although it is the period of musical scales and sonorities, Tetora exhibits evidence of a timepoint structure whose compositional design and its application relate to transformations parallel to his 'metabolae'. His theory would generate a way of thinking about pitch structures and sonorities, that as the analysis of Tetora shows - was also to be extended to the temporal structure of (certain parts of) compositions.


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Tetora (1990, for string quartet) was written in the middle of a period characterised by the application of sieves in pitch-space (creation of scales). Xenakis conceived Sieve Theory in 1963 when he was named artist in residence in West Berlin. ${ }^{1}$ The theory's initial aim was the axiomatization of musical structures. It was used in the compositional method for some of the works of the 60 's and, more frequently, after 1977. The two fundamental criteria of the theory are symmetry and periodicity. At first, sieves would appear in the form of residue class sets, which the composer would manipulate using the logical operations of union (+), intersection (*), and complementation ( - ). Sieves, in their (set-) theoretical form, would be constructed and transformed before their transcription into the form they would occupy in the compositional end-result. However, this method of formalizing musical structures was to be abandoned by the composer in the second phase of the theory's application (mid 80 's - mid 90 's). In order to illustrate this re-application, we are going to consult the 'reconstruction' of sieves that Benoît Gibson suggests [see 2: 56-7].

Sieves can be distinguished according to the two criteria of symmetry and periodicity (internal and external symmetry):

- periodic symmetric sieve

It is the union of two or more residue class sets. e.g. $3_{2}+4_{2}=\{2,5,6,8,10,11,14, \ldots\}$. The period of this sieve is the smallest common multiple of its factors (12). The sieve's internal symmetry is indicated by its intervallic structure: 312213 . This is the case with Messiaen's non-retrogradable rhythms.

- periodic asymmetric sieve

The theoretical representation of a periodic asymmetric sieve is only possible according to its period. Its intervallic structure is asymmetric. This is the case of the major scale: $12_{0}+12_{2}+12_{4}+$ $12_{5}+12_{7}+12_{9}+12_{11}{ }^{2}$

Symmetric sieves are the simplest ones - a kind that Xenakis had never used in his compositions. On the contrary, his sieves in the 60 's would be of the second type. In the 80 's the composer would be preoccupied with non-periodic asymmetric sieves. It is a case of a complete reversal in relation to the initial conception of sieves; an elimination of the two fundamental criteria. Sieves are now constructed according to other criteria. Xenakis himself refers, in his Conversations with Bálint Andárs Varga, to the notion of tension, which is to be arrived at through
the opposition of large and small intervals - that is, the contrast between something very narrow and something much larger. To maintain this tension all along the sieve - in other words in the scale you have chosen - is a tall order. It is also an intriguing problem: none of the parts is to be symmetric - that is, periodic; nor are the ranges to be periodic as compared to the higher or lower ranges, maintaining tension all the while in a different way [4: 146].

Xenakis mentions this in a discussion of his choral work $\operatorname{Orkos}$ (1981) in reference to the notion of the scale, and the influence of Javanese music - more specifically of the 'pelog' scale - that he had considered highly important in his music. ${ }^{3}$

This is the type of the non-periodic asymmetric sieve that the following discussion will be concerned with. Before we go on, we will have to consider the role of sieves in this second phase of the theory. It must be clear by now that sieves and scales are now identical. ${ }^{4}$ After all, this had been a major aim of the theory's initial use (Xenakis offered an illustration of his theory using the examples of the major scale, of Byzantine and Indian scales in combination with the Aristoxenean segments) [see 5: 197-8]. Makis Solomos states that clearly, saying

[^0]because they are scales, sieves are directly connected to the problematics of pitch; and their generalization carries its trace. The 'fundamental' scope of sieves therefore remains limited (to the domain of pitch) [3: 89].

The sieve of Tetora that we are going to discuss is an exception, a deviation from the rule of sieves as scales. It comprises a rhythm, a time-point sequence that is located in the several chord sequences throughout the quartet:

## 311311231211322211112322221112

where the unit of elementary displacement is the semiquaver.


Example 1: Tetora, bars 40-42
The rhythmic sieve is initially located in bars 21-24. It is the rhythm of a chord sequence, whose chords are only provisionally considered as such, since they are merely tetrachords consisting of successive elements of a pitch sieve. In all the appearances of the chord sequence (apart from the first one) there is a straight-forward layering of the quartet in the upper and lower strings (see Example 1). It is the initiation of a system of transformations that, as it appears in Xenakis's sketches, consists of eight chord sequences with their corresponding time-point sequences. For reasons that will become clear later on, we will examine the first three sequences in comparison. In the first part of Table 1, the upper-case Latin characters stand for the tetrachords of successive elements of the same (pitch-) sieve ( $\mathbf{X}$ stands for the lower chord of bars 21-24); underlined characters refer to the lower strings. Lower case Greek characters stand for chords that belong to the chord set of Example 2. The intervallic structure of the rhythmic sieve (INT) is stated in grey numbers under the actual chord sequence, and under the bar numbers is stated the transposition of the chords in relation to their first appearance in the work, in bars 38-39 (the index refers to semitones, so $\mathrm{T}_{0}$ means that the chord has not been transposed).

There is a process of gradual specification that takes place on two levels: a) the chords, first in the lower strings of the second sequence and then in all strings of the third sequence, belong in the chord set of Example 2; b) the process of chord substitution from the second to the third sequence is consistent. ${ }^{5}$ From now on, all the chords in the sequences will either belong to the set of Example 2, or to that of Example 3. During this process we have the first realization in the work of the outside-time structure that is the rhythmic sieve, as the rhythmic structure of all the sequences in the work. Xenakis's theory of outside-time musical structures, as illustrated in [5], would have remained fragmented without a process that would enable the setting out of a design of the inside-time structure of the composition. Let this be the definition of the Xenakian 'metabola' [see 5: 190].

Metabolae (transformations) in Tetora are of three types: a) substitution of time-intervals, b) chord substitution, and c) chord transposition. (From these, only the second took place in the first phase of the process of metabolae - Table 1). The second phase of metabolae appears in bars 59-85, as shown in Table 2, which exhibits certain anomalies in its structure. ${ }^{6}$ Part (a) consists of two sections:

[^1]a) Chord sequences and transpositions:

b) Chord sequences and substitutions:
$$
40-42 \rightarrow 48-50
$$
\[

$$
\begin{aligned}
& \mathbf{F} \rightarrow \mathbf{s} \\
& \mathbf{C} \rightarrow \underline{\delta} \\
& \mathbf{E} \rightarrow \underline{\varepsilon} \\
& \mathbf{A} \rightarrow \boldsymbol{\gamma}
\end{aligned}
$$
\]

Table 1: Bars 21-24, 40-42, and 48-50

a) Chord sequences and transpositions:

|  | $\begin{gathered} \text { Bars 59-62 } \\ \left(T_{-2}\right) \end{gathered}$ | chord INT | $\varepsilon$ | $\begin{aligned} & \gamma \\ & 2 \end{aligned}$ | $\frac{\varepsilon}{2}$ | $\boldsymbol{\varepsilon}$ 1 | $\begin{aligned} & \gamma \\ & 2 \end{aligned}$ | $\underline{\underline{\delta}}$ | $\boldsymbol{\delta}$ | $1$ | $\gamma$ 2 |  | $\bigcirc$ | $\gamma$ 2 | $\underset{2}{ }$ | $\frac{\varepsilon}{1}$ | ¢ 3 | ¢ 3 | E 3 | $\frac{\text { ¢ }}{2}$ | ${ }_{2}$ | $\begin{aligned} & \boldsymbol{\varepsilon} \\ & 2 \end{aligned}$ | $\frac{\varepsilon}{2}$ | ¢ 3 | $\varepsilon$ | $\delta$ | $\varepsilon$ | $\begin{aligned} & \gamma \\ & 3 \end{aligned}$ | $\varsigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Bars 62-65 | chord | $\delta$ | ¢ | $\underline{\delta}$ | $\delta$ | $\varsigma$ | $\bigcirc$ | $\beta$ | $\delta$ | $\varsigma$ |  | $\underline{\delta}$ | ¢ | $\varepsilon$ | $\underline{\varepsilon}$ | $\boldsymbol{\beta}$ | $\beta$ | $\boldsymbol{\delta}$ | $\underline{\varepsilon}$ | $\varsigma$ | ס | $\underline{\delta}$ | $\beta$ | $\delta$ | $\beta$ | $\delta$ | $\varsigma$ | $\varsigma$ |
|  | ( $\mathrm{T}_{+3}$ ) | INT | 2 | 3 | $\underline{3}$ | 2 | 3 | $\underline{3}$ | 1 | 2 | 3 |  | 1 | 3 | 3 | $\underline{2}$ | 1 | , | 1 | $\underline{3}$ | , | 3 | $\underline{3}$ | , | 2 | 1 | 1 | 1 |  |
|  | Bars 65-68 | chord | $\boldsymbol{\alpha}$ | \& | ¢ | $\boldsymbol{\alpha}$ | $\varepsilon$ | $\underline{\varepsilon}$ | $\gamma$ | $\alpha$ | $\varepsilon$ |  | $\underline{\underline{\delta}}$ | $\varepsilon$ | $\varsigma$ | S | $\gamma$ | $\gamma$ | $\boldsymbol{\alpha}$ | $\underline{\varepsilon}$ | $\varepsilon$ | $\boldsymbol{\alpha}$ | ¢ | $\gamma$ | $\boldsymbol{\alpha}$ | $\gamma$ | $\alpha$ | $\varepsilon$ | $\underline{\delta}$ |
|  | ( $\mathrm{T}_{-1}$ ) | INT | 2 | 2 | $\underline{2}$ | 2 | 2 | $\underline{2}$ | 1 | 2 | 2 |  | 1 | 2 | 2 | $\underline{2}$ | 1 | 1 | 1 | $\underline{2}$ | 2 | 2 | $\underline{2}$ | 1 | 2 | 1 | 1 | 1 |  |
|  | Bars 69-71 | chord | $\gamma$ | $\boldsymbol{\beta}$ | $\varsigma$ | $\gamma$ | $\boldsymbol{\beta}$ | $\varsigma$ | $\alpha$ | $\gamma$ | $\beta$ |  | $\bigcirc$ | $\beta$ | $\delta$ | $\varsigma$ | $\alpha$ | $\alpha$ | $\gamma$ | $\bigcirc$ | $\beta$ | $\gamma$ | ¢ | $\alpha$ | $\gamma$ | $\boldsymbol{\alpha}$ | $\gamma$ | $\beta$ | $\bigcirc$ |
|  | ( $\mathrm{T}_{+1}$ ) | INT | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | $\underline{1}$ | 1 | 1 | 1 | $\underline{1}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\zeta$ | Bars 71-73 | chord | $\boldsymbol{\beta}$ | $\boldsymbol{\alpha}$ | $\underline{\varepsilon}$ | $\beta$ | $\alpha$ | $\bigcirc$ | $\varsigma$ | $\boldsymbol{\beta}$ | $\alpha$ |  | $\bigcirc$ | $\boldsymbol{\alpha}$ | $\varepsilon$ | $\underline{\varepsilon}$ | $\varsigma$ | $\varsigma$ | $\boldsymbol{\beta}$ | S | $\alpha$ | $\boldsymbol{\beta}$ | $\underline{\varepsilon}$ | 5 | $\beta$ | $\bigcirc$ | $\beta$ | $\boldsymbol{\alpha}$ | $\underline{\varepsilon}$ |
|  | ( $\mathrm{T}_{-3}$ ) | INT | 2 | 1 | $\underline{2}$ | B | 2 | 1 | 2 | 1 | 2 |  | 1 | 2 | 1 | $\underline{2}$ | 1 | 2 | 1 | $\underline{2}$ | 1 | 2 | $\underline{2}$ | 2 | 1 | 2 | 1 | 2 |  |
|  | Bars 73-74 | chord |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\boldsymbol{\alpha}$ | $\underline{\varepsilon}$ | $\varsigma$ | $\boldsymbol{\alpha}$ | S | $\boldsymbol{\beta}$ | $\alpha$ | $\beta$ | $\alpha$ | ¢ | $\varsigma$ |
|  | ( $\mathrm{T}_{+2}$ ) | INT |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | $\underline{2}$ | , | 2 | 1 | 3 | 1 | 2 | 1 | 2 |  |
|  | Bars 74-76 | chord | $\gamma$ | $\gamma$ | $\underline{\text { ¢ }}$ | $\gamma$ | $\gamma$ |  | $\gamma$ | $\gamma$ | $\gamma$ |  | $\underline{\delta}$ | $\gamma$ | $\gamma$ | $\underline{\varepsilon}$ | $\gamma$ | $\gamma$ | $\gamma$ | S | $\gamma$ | $\gamma$ | $\underline{\delta}$ | $\gamma$ | $\gamma$ | $\gamma$ | $\gamma$ | $\gamma$ | $\underline{\varepsilon}$ |
|  | $\left(\mathrm{T}_{+2}\right)$ | INT | 1 | 1 | $\underline{1}$ | 1 | 1 | $\underline{1}$ | , | 1 | 1 |  | $\underline{1}$ | 1 | r | 1 | , | , | , | 1 | , | , | 1 | , | 1 | , | 1 | r |  |
|  | Bars 83-85 | chord | $\alpha$ | $\varsigma$ |  | $\alpha$ | $\leqslant$ | $\underline{\delta}$ | $\beta$ | $\alpha$ |  | ( $\alpha$ ) |  | $\varsigma$ | $\delta$ |  | $\beta$ | $\beta$ | $\alpha$ |  |  | $\alpha$ | S | $\beta$ | $\alpha$ | $\beta$ | $\alpha$ |  | $\varsigma$ |
|  | ( $\mathrm{T}_{+4}$ ) | INT | 3 | 1 | $\underline{2}$ | 1 | 3 | 1 | 2 | 1 | 2 | 1 | 1 | 2 | 1 | $\underline{2}$ | 1 | 3 | 1 | $\underline{2}$ | 1 | 2 | 1 | 3 | 1 | 2 | 1 | 2 |  |
|  | $\begin{gathered} \text { Bars 83-85 } \\ \left(\mathbf{T}_{+4}\right) \end{gathered}$ | chord | [ 5 | $\varsigma$ | $\boldsymbol{\alpha}$ | $\beta$ | $\boldsymbol{\alpha}$ | $\beta$ | ¢ | $\boldsymbol{\alpha}$ | $\varsigma$ | $\varepsilon]^{7}$ | $\alpha$ | $\beta$ | $\beta$ | S | $\boldsymbol{\delta}$ | $\varsigma$ | $\underline{\varepsilon}$ | $\alpha$ | $\varsigma$ | $\boldsymbol{\alpha}$ | $\beta$ | $\underline{\delta}$ | $\varsigma$ | $\boldsymbol{\alpha}$ |  |  | $\alpha$ |
|  |  | INT | $\underline{1}$ | 1 | 2 | 1 | 3 | 1 | $\underline{2}$ | 1 | 2 | 11 | 3 | 1 | 2 | 1 | 2 | 1 | 1 | 2 | 1 | 2 | 1 | $\underline{3}$ | 1 | 2 | $\underline{1}$ | 3 |  |

b) Chord sequences and substitutions:

| 59-62 |  | 62-5 |  | 65-8 |  | 69-71 |  | 71-3 |  | $\begin{array}{r} 73-4 \\ (83-5 \end{array}$ |  | 74-6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\varepsilon$ | $\rightarrow$ | $\delta$ | $\rightarrow$ | $\boldsymbol{\alpha}$ | $\rightarrow$ | $\gamma$ | $\rightarrow$ | $\beta$ | $\rightarrow$ | $\alpha$ | $\rightarrow$ | $\gamma$ |
| $\gamma$ | $\rightarrow$ | $\varsigma$ | $\rightarrow$ | $\varepsilon$ | $\rightarrow$ | $\beta$ | $\rightarrow$ | $\alpha$ | $\rightarrow$ | $\varsigma$ | $\rightarrow$ | $\gamma$ |
| $\delta$ | $\rightarrow$ | $\beta$ | $\rightarrow$ | $\gamma$ | $\rightarrow$ | $\alpha$ | $\rightarrow$ | $\varsigma$ | $\rightarrow$ | $\beta$ | $\rightarrow$ | $\gamma$ |
| $\beta$ | $\rightarrow$ | $\varepsilon$ | $\rightarrow$ | $\varsigma$ | $\rightarrow$ | $\delta$ | $\rightarrow$ | $\varepsilon$ | $\rightarrow$ | $\delta$ | $\rightarrow$ | $\gamma$ |
| 1 | $\rightarrow$ | 2 | - | 2 | $\rightarrow$ | 1 | - | , | - | 1 | - | 1 |
| 2 | $\rightarrow$ | 3 | $\rightarrow$ | 2 | $\rightarrow$ | 1 | $\rightarrow$ | 2 | $\rightarrow$ | 3 | $\rightarrow$ | 1 |
| 3 | $\rightarrow$ | 1 | - | 1 | - | 1 | - | 1 | $\rightarrow$ | $2^{8}$ | $\rightarrow$ | 1 |

Table 2: Bars 59-68, 69-76, and 83-

[^2]${ }^{8}$ With an exception at the correspondence of the last chord of bar 72 with the second from the end of bar 83 (1 semiquaver instead of 2 or 3 ).
in the first (bars 59-68) there are three sequences in succession. In the second section (bars 69-76) there are certain anomalies in the transition from one sequence to the next. It is an incomplete realization (through a process of metabolae) of a compositional design that we will here demonstrate. This imperfect realization can be seen as follows: from the four sequences, one (bar s73-74) appears only partially. In the table, its role is shown by relocating it in paradigmatic association to the other sequences. (At this point we will have to note that our examination of how this associative process is carried out ignores the lower strings, as there can be found no systematic application of any kind in the process of chord substitution. The criterion, therefore, is the systematic application of a design of chord substitution in the upper strings; this association is also aided by the actual structure of the sequence, in the way the upper strings are been 'interrupted' by the lower).

Seven bars after the completion of the sequence in bars 74-76, there is another one which appears temporally isolated from the rest (bars 83-85), and whose relation to the process of metabolae is not straight-forward (in the table it is found at the bottom of part [a]). This is due to two reasons: firstly, its isolation (actually, its displacement) and secondly, its very structure, which is the retrograde of the structure of the rest. It remains to discover its role in relation to the metabolae, first to the process of chord substitution and later to the one of substitutions of time-intervals - which we will include in our discussion later on. Close inspection will reveal that the incomplete sequence of bars 7374 is actually found, in retrograde form, at the beginning of the one in bars 83-85. Therefore, if the retrograde form of the sequence in bars $83-85$ is the reason to present it (in the table) in its 'prime' form, its identification with the incomplete sequence of bars 73-74 is the reason to (re)locate it in the latter's position. This will enable us to indicate the discrepancy that has been caused (with the displacement and inversion) in the process of metabolae that does not appear on the surface.

Part (b) of the table shows the dual process of substitution - of chords and time-intervals. While part (a) could be characterised 'paradigmatic', part (b) is more 'syntagmatic': the columns of the former now appear as the rows of the latter. Its horizontal dimension now symbolizes time and can be read from left to right throughout as a continuous process of substitutions. Examining, for now, the chord substitutions we can interpret the correspondent part of the table as a synoptic chart of this second phase of metabolae.


Example 3: Chords of bars 115-137
In Table 2 each of the four chords of each sequence is gradually substituted for another four, apart from one chord (fourth row of part [b]) which is substituted by three. In bars 74-76 the whole procedure ends up in the simple repetition of a single chord. Thus, the overall homogeneous nature of the process yields to a sequence's maximally non-differentiated structure, a sequence which also exhibits a non-differentiated rhythmic structure (the steady pulse of a semiquaver). It is a matter of convergence of two dimensions - of the horizontal, that is the internal structure of the chord sequence, and of the vertical, the process of chord substitutions itself. In the table this can be seen as the convergence of the 'paradigmatic' and 'syntagmatic' dimensions (the row of [a] and the column of [b] that correspond to bars 74-76). It is exactly this collapse of homogenous variety to the nondifferentiated that has been prevented by the temporal displacement (and distortion) of a part of the process (bars 83-85). It is a matter of the very same aesthetic criterion of tension at a more distant level of focusing in time: a breakdown of regularity. It is not by mere chance that in the sequence of bars 8385 the chords that belong to the set of Example 2 sound for the last time, reminiscent of what has already concluded. From now on, we are transferred to the tonal universe of Example 3, in bars 115137, which we are going to examine straight away.

## a) Chord sequences and transpositions:

| $\begin{gathered} \text { Bars 115-116 } \\ \left(\mathrm{T}_{0}\right) \end{gathered}$ | chord INT | $\begin{gathered} \mathbf{\delta} \\ 1 \end{gathered}$ | $\begin{gathered} \boldsymbol{\varepsilon}^{\prime} \\ 1 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\begin{gathered} \boldsymbol{\delta} \\ 1 \end{gathered}$ | $\begin{gathered} \boldsymbol{\varepsilon}^{\prime} \\ 1 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\begin{aligned} & \varsigma^{\prime} \\ & 1 \end{aligned}$ | $\begin{gathered} \mathbf{\delta} \\ 1 \end{gathered}$ | $\begin{gathered} \boldsymbol{\varepsilon}^{\prime} \\ 1 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\begin{gathered} \boldsymbol{\varepsilon}^{\prime} \\ 1 \end{gathered}$ | $\begin{gathered} \gamma \\ 1 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\begin{aligned} & \varsigma^{\prime} \\ & 1 \end{aligned}$ | $\begin{aligned} & \xi^{\prime} \\ & 1 \end{aligned}$ | $\begin{gathered} \boldsymbol{\delta} \\ 1 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\begin{gathered} \boldsymbol{\varepsilon}^{\prime} \\ 1 \end{gathered}$ | $\begin{gathered} \boldsymbol{\delta} \\ 1 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\begin{aligned} & \varsigma^{\prime} \\ & 1 \end{aligned}$ | $\begin{gathered} \mathbf{\delta} \\ 1 \end{gathered}$ | $\begin{aligned} & \xi^{\prime} \\ & 1 \end{aligned}$ | $\begin{gathered} \mathbf{\delta} \\ 1 \end{gathered}$ | $\begin{aligned} & \boldsymbol{\varepsilon}^{\prime} \\ & 1 \end{aligned}$ | $\underline{\varepsilon}{ }^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Bars 116-119 } \\ \left(\mathrm{T}_{+1}\right) \end{gathered}$ | chord INT | $\begin{aligned} & \xi^{\prime} \\ & 2 \end{aligned}$ | $\underset{2}{\boldsymbol{\beta}}$ | $\frac{\varepsilon^{\prime}}{\underline{2}}$ | $\begin{aligned} & \prime \\ & 2 \end{aligned}$ | $\underset{2}{\boldsymbol{\beta}}$ | $\frac{\delta^{\prime}}{\underline{2}}$ | $\begin{gathered} \mathbf{\delta} \\ 1 \end{gathered}$ | $\begin{aligned} & \xi^{\prime} \\ & 2 \end{aligned}$ | $\underset{2}{\boldsymbol{\beta}}$ | $\frac{\gamma^{\prime}}{\underline{1}}$ | $\beta$ | $\alpha^{\prime}$ | $\frac{\varepsilon^{\prime}}{\underline{2}}$ | $\delta^{\prime}$ | $\delta^{\prime}$ | $\begin{aligned} & \varsigma^{\prime} \\ & 1 \end{aligned}$ | $\frac{\delta^{\prime}}{\underline{2}}$ | $\begin{gathered} \boldsymbol{\beta} \\ 2 \end{gathered}$ | $\begin{aligned} & \prime \\ & 2 \end{aligned}$ | $\frac{\varepsilon^{\prime}}{\underline{2}}$ | $\delta^{\prime}$ | $\begin{aligned} & \prime \\ & 2 \end{aligned}$ | $\delta$ | $\begin{gathered} \xi^{\prime} \\ 1 \end{gathered}$ | $\boldsymbol{\beta}^{\prime}$ | $\chi^{\prime}$ |
| $\begin{aligned} & \text { Bars 119-122 } \\ & \left(T_{-1}\right) \end{aligned}$ | chord INT | $\begin{aligned} & \boldsymbol{\gamma} \\ & 2 \end{aligned}$ | $\begin{gathered} \boldsymbol{\alpha} \\ 3 \end{gathered}$ | $\begin{aligned} & y^{\prime} \\ & \underline{3} \end{aligned}$ | $\begin{aligned} & \gamma \\ & 3 \end{aligned}$ | $\begin{gathered} \boldsymbol{a} \\ 3 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{3}}$ | $\begin{gathered} \boldsymbol{\varepsilon} \\ 1 \end{gathered}$ | $\begin{aligned} & \boldsymbol{\gamma} \\ & 2 \end{aligned}$ | $\begin{gathered} \boldsymbol{\alpha} \\ 3 \end{gathered}$ | $\begin{aligned} & \gamma^{\prime} \\ & \underline{1} \end{aligned}$ | $\alpha$ | $\begin{gathered} \boldsymbol{\beta} \\ 3 \end{gathered}$ | $\frac{\underline{\delta}^{\prime}}{\underline{2}}$ | $\begin{gathered} \varepsilon^{\prime} \\ 1 \end{gathered}$ | $\begin{gathered} \varepsilon^{\prime} \\ 1 \end{gathered}$ | $\begin{gathered} \boldsymbol{\gamma} \\ 1 \end{gathered}$ | $\frac{\delta^{\prime}}{\underline{3}}$ | $\alpha$ | $\begin{gathered} \boldsymbol{\gamma} \\ 3 \end{gathered}$ | $\gamma^{\prime}$ | ( $\varepsilon^{\prime}$ ) | $\left(\gamma^{\prime}\right)$ |  | ( $\varepsilon^{\prime}$ ) | $\left(\gamma^{\prime}\right)$ | ( $\alpha^{\prime}$ ) |
| $\begin{gathered} \text { Bars 122-125 } \\ \left(\mathrm{T}_{+3}\right) \end{gathered}$ | chord <br> INT | $\begin{gathered} \mathbf{\alpha} \\ 3 \end{gathered}$ | $\begin{aligned} & \gamma \\ & 1 \end{aligned}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\begin{gathered} \boldsymbol{\alpha} \\ 3 \end{gathered}$ | $\begin{gathered} \gamma \\ 1 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{1}}$ | $\underset{2}{\boldsymbol{\beta}}$ | $\boldsymbol{\alpha}$ 3 |  |  |  | ס' | $\underline{\underline{8}}{ }^{\prime}$ | ${ }^{\boldsymbol{\beta}} 2$ |  | $\boldsymbol{\alpha}$ 2 | $\frac{\varepsilon^{\prime}}{\underline{1}}$ |  | $\boldsymbol{\alpha}$ 1 | $\frac{\delta^{\prime}}{\underline{2}}$ |  | $\boldsymbol{\alpha}$ 1 |  | $\begin{gathered} \boldsymbol{\beta} \\ 1 \end{gathered}$ | $\begin{gathered} \boldsymbol{\alpha} \\ 2 \end{gathered}$ | $\begin{aligned} & \boldsymbol{\gamma} \\ & 2 \end{aligned}$ |
| $\begin{aligned} & \text { Bars 125-128 } \\ & \left(T_{-2}\right) \end{aligned}$ | chord INT | $\underset{\sim}{\boldsymbol{\beta}}$ | $\begin{gathered} \mathbf{\delta} \\ 2 \end{gathered}$ | $\frac{\delta^{\prime}}{\underline{2}}$ | $\underset{1}{\boldsymbol{\beta}}$ | $\begin{gathered} \mathbf{\delta} \\ 2 \end{gathered}$ | $\frac{\gamma^{\prime}}{\underline{2}}$ | $\begin{gathered} \boldsymbol{\gamma} \\ 3 \end{gathered}$ | $\begin{gathered} \boldsymbol{\beta} \\ 1 \end{gathered}$ | $\begin{gathered} \boldsymbol{\delta}^{\prime} \\ 2 \end{gathered}$ | $\frac{\varepsilon^{\prime}}{\underline{3}}$ | $\begin{gathered} \mathbf{\delta} \\ 2 \end{gathered}$ | $\begin{gathered} \boldsymbol{\varepsilon} \\ 2 \end{gathered}$ | $\frac{\delta^{\prime}}{\underline{1}}$ | $\begin{gathered} \boldsymbol{\gamma} \\ 3 \end{gathered}$ | $\begin{aligned} & \gamma \\ & 3 \end{aligned}$ | ${ }^{\boldsymbol{\beta}}$ | $\frac{\gamma^{\prime}}{\underline{2}}$ | $\begin{gathered} \mathbf{\delta} \\ 2 \end{gathered}$ | ${ }_{2}$ | $\frac{\delta^{\prime}}{\underline{2}}$ | ( $\gamma^{\prime}$ ) <br> (3) | $\begin{gathered} \boldsymbol{\beta} \\ 1 \end{gathered}$ |  | $\begin{aligned} & \boldsymbol{\gamma} \\ & 3 \end{aligned}$ | $\begin{gathered} \boldsymbol{\beta} \\ 3 \end{gathered}$ | $\begin{gathered} \mathbf{\delta} \\ 3 \end{gathered}$ |

## b) Chord sequences and substitutions:

| 115-116 |  | 116-119 |  | 119-122 |  | 122-125 |  | 125-128 ${ }^{10}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta^{\text {, }}$ | $\rightarrow$ | $\varsigma^{\prime}$ | $\rightarrow$ | $\gamma^{\prime}$ | $\rightarrow$ | $\boldsymbol{\alpha}$ | $\rightarrow$ | $\beta$, |
| $\varepsilon$ | $\rightarrow$ | $\beta$, | $\rightarrow$ | $\boldsymbol{\alpha}$ | $\rightarrow$ | $\gamma^{\prime}$ | $\rightarrow$ | $\delta$ ' |
| $\varsigma^{\prime}$ | $\rightarrow$ | $\boldsymbol{\delta}$ ' | $\rightarrow$ | $\varepsilon$ ' | $\rightarrow$ | $\boldsymbol{\beta}$ | $\rightarrow$ | $\gamma$ ' |
| $\gamma$, | $\rightarrow$ | $\boldsymbol{\alpha}$ | $\rightarrow$ | $\boldsymbol{\beta}$ | $\rightarrow$ | ${ }^{\text {, }}$ | $\rightarrow$ | ¢ |
| 1 | $\rightarrow$ | 2 | - | 2 | $\rightarrow$ | 3 | $\rightarrow$ | 1 |
| 1 | $\rightarrow$ | 2 | $\rightarrow$ | 3 | $\rightarrow$ | $1^{11}$ | $\rightarrow$ | 2 |
| 1 | - | 1 | - | 1 | $\rightarrow$ | 2 | $\rightarrow$ | 3 |

Table 3: Bars 115-128

[^3]Table 3 is constructed exactly as its preceding one and shows the substitutions and transpositions of bars 115-128. Now all the sequences are following one another in direct succession and the metabolae are straight-forward. Comparing the chord substitutions in part (b) of Tables 2 and 3 , we notice the transition to an absolutely regular and homogeneous process. In bars 59-85 we saw a more elaborate application a compositional system, whose characteristics had already started to appear in bar 40. This system will be eventually crystallized in bars 115-128. ${ }^{12}$ Five sequences, each containing four chords, will finally allow regularity be established. It is a process which concludes in absolute homogeneity, as if permitting the appearance of different sides of a symmetric object: four chords are been substituted four times (in part [b] of Table 3 each column has four chords and each row has five). However, the end of this process will not signify the end of the work; it will end with three more sequences (bars 128-138) where the two duos, in two-part counterpoint, will present in irregular order the chords of Example 3, based on the initial form of the rhythmic sieve.

The aforementioned rhythmic sieve manifests the aesthetic criterion of tension through metabolae that, although not parallel to the chord substitutions, in certain parts do coincide. This sieve, as a generalization of pitch sieves, carries their 'trace'. There is a clear sense of correspondence, in the work, between the interval of the semitone and the time-interval of the semiquaver as the common unit of elementary displacement. The notion of tension, although far from being objective, is actually present in Tetora, as the juxtaposition of small and large intervals, and this is audible in the rhythm of the chord sequences. In pitch sieves, the free interchange of the sounds themselves, that is of sounds that belong to a region in the scale, will suffice to produce tension, which is eventually produced by the intervallic structure of the sieve itself (without having to resort to melodic patterns). In a scale, it is generally possible to have a succession of pitches from different, distant regions. In that sense there could be no equivalence between pitch and time-points. However, while 'in chromatic and welltempered scales you can generate tension only through jumps, as in serial music' [4: 145], Xenakian sieves are such constructed that the succession of neighbour elements is necessary in order for the structure of sieves to be revealed or, better, to 'function'. Thus, we can speak both of neighbouring (of tension) in space and in time - the idea of intervallic structure is now closer to that of prosody.

Xenakis transforms the rhythmic sieve of Tetora selectively applying a permutation of (time-) values in order to either maintain tension, or allow a transition from tension to relaxation and vice versa. A three-element set has six possible permutations $(3!=6)$. Whenever there is a permutation in Tetora, it corresponds to one of the three positions of the rotation of the triangle. This process is applied in certain parts of the work and each new position gives a new sieve, a new position of the sieve, which exhibits the same degree of differentiation in its structure. The process of metabolae in the rhythmic sieve throughout the quartet is shown in Table 4 (the structure of this table is based on that of the second part of the chord substitution tables). Practically, permutation means to correspond each element of a set to a different element of itself. Rotation, as a special case of permutation, is the symmetric procedure which is employed in order to help maintain tension in each new form (position) of the sieve. This tension is neutralized only when rotation ceases to happen. Two observations concerning the convergence of this process with that of chord substitution: The first, as has already been mentioned, refers to the convergence on the maximally non-differentiated structure of the chord sequence in bar 74 and its rhythmic structure - the persistent repetition (in the upper strings) of a single chord. The second concerns the sequence of bars 83-85 that appears (partially and transformed) before and after the one in bar 74. Its unique role (i.e., breakdown of regularity) is also apparent in the structure of the rhythmic sieve itself: it is the only time in the work when the appearance of all three time-values in the sieve has not resulted from rotation (neither from any other kind of function). With its maximally differentiated structure it arbitrarily intervenes between two totally regular sieves (onesemiquaver pulses).

| bars: | 21 1 | 40 $1{ }^{\text {st }}$ | 48 18 | 59 $2^{\text {nd }}$ | $3^{62}$ | 65 | 69 | 71 | $\begin{gathered} 73 \\ (83) \end{gathered}$ | 74 | 115 | 116 | 119 3 | 122 $1{ }^{\text {st }}$ | 125 $2^{\text {nd }}$ | 128 $1{ }^{\text {st }}$ | 131 1 | 134 $1^{\text {st }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 | 3 | 3 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 1 | 3 | 3 | 3 |
|  | 1 | 1 | 1 | 2 | 3 | 2 | 1 | 2 | 3 | 1 | 1 | 2 | 3 | 1 | 2 | 1 | 1 | 1 |
|  | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 2 | 3 | 2 | 2 | 2 |

Table 4: Rotations of the triangle in Tetora

[^4]From bar 128 to the end, all the chords in the set of Example 3 appear in irregular order in both the upper and lower strings. The final sequence (from bar 134 to the end) will negate apparent repetition, by presenting the rhythmic sieve in its retrograde form. This is the only time in the work, when any of the positions of the sieve (in this case the first) appears in this form. (We remind ourselves that in the previous instant of retrograde form, the rhythmic sieve did not derive from the initial). Xenakis transforms the sieve for a last time, in a way that has not been encountered in the works so far - it is neither a repetition of a process (permutation), nor of a chord sequence. In the same manner that Xenakis negated symmetry in the sieves of his later period, he will employ the symmetric process of the rotation of the triangle, in order afterwards to negate it (see Table 4, bars 65-119). It is not a fundamental criterion anymore but a tool (as, after all, every theory is) in the hands of the composer. The retrograde form, as another symmetry, yields another form of the sieve, which (as in its initial appearance) manifests and maintains tension for the last time.

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[^0]:    ${ }^{1}$ His research at that time, led him (among other things) to encounter what in mathematics in known as the 'Sieve of Eratosthenes' - in mathematics, a schema for determining the prime numbers between two given integers [see 1: 39-40].
    ${ }^{2}$ A factorial representation of such sieves (in the way, that is, we did with symmetric sieves) would have to employ the logical operation of intersection.
    ${ }^{3}$ The first work where such a sieve was used is Jonchaies (1977, for orchestra) [See 3: 90].
    ${ }^{4}$ Unlike the rhythmic sieves in the works (among others) for percussion of the 60 's, like e.g. Persephassa (1969, for six percussionists).

[^1]:    ${ }^{5}$ In the chord sequence of bars 48-50 there is a reversal, where the lower strings take the role of the upper, and the other way around. See Table 1.
    ${ }^{6}$ An additional chord, in the tenth column of the table, appears in brackets $-(\boldsymbol{\alpha})$. The space between the third and fourth rows of the table suggests that other material intervenes between the corresponding chords sequences.

[^2]:    ${ }^{7}$ Retrograde of bars 73-4.

[^3]:    ${ }^{9}$ With one exception: in chord $\boldsymbol{\alpha}$ ' C 6 has been substituted for $\mathrm{D} \# 6$, where C 4 is middle C .
    ${ }^{11}$ With some exception after the interruption of the normal evolution of the sequences (See the last six columns of part [a] of the table).
    ${ }^{11}$ With an exception at the correspondence of the last chord of bar 119 with the fourth of bar 122 (3 semiquavers instead of 2).

[^4]:    ${ }^{12}$ This second instantiation of the system is not found as such in the sketches to Tetora. However, it clearly follows the same rationale.

